

Linear algebra section 2

Yashi

January 2026

Reduced Row Echelon Form (RREF)

Definition

A matrix is said to be in *reduced row echelon form (RREF)* if it satisfies all of the following:

1. All nonzero rows appear above any rows consisting entirely of zeros.
2. The leading entry (pivot) of each nonzero row is equal to 1.
3. Each leading 1 is the only nonzero entry in its column.
4. The leading 1 of each nonzero row occurs strictly to the right of the leading 1 in the row above.

Problems: Is the Matrix in RREF?

Instructions. For each matrix below, determine whether it is in reduced row echelon form. If not, state which condition of the definition fails.

1

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

2

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4

$$\begin{pmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

Problem 5 (Answer in yes or no) Can a matrix that is **not** upper triangular be in its rref form?

Problem 6

Convert the following system of equations into an augmented matrix and find its reduced row echelon form. Hence determine the solution set.

$$x_3 + x_4 = 0,$$

$$x_2 + x_3 = 0,$$

$$x_1 + x_2 = 0,$$

$$x_1 + x_4 = 0.$$

Problem 7

For which values of a, b, c, d , and e is the following matrix in reduced row-echelon form?

For which values of a, b, c, d , and e is the following matrix in reduced row-echelon form?

$$A = \begin{pmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{pmatrix}$$