

Linear algebra (Spring 2026) week 1

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Activity 1 (Sneaky mathematicians) You are in $\mathbb{R}^2$ and situated at the point $(0,0)$. You have a plane that can fly in any direction you want.

1. Mathematician Newton is hiding in the line $y = 7x$. Choose a direction (i.e. vector) you would want your plane to fly in (starting at the origin) so that you can find Newton. You can't change the direction of the plane once chosen.
2. Mathematician Gauss is hiding somewhere in the line $y = 7x - 23$. You now have a hoverboard that can transport you to any point p in $\mathbb{R}^2$. Choose a point $\vec{p} = (x, y)$ and a direction (ie a vector $\vec{v}$) that your plane has to fly in so that you can find Gauss. You can't change the direction once chosen. If t is the time the plane needs to travel from point p to find Gauss, can you find coordinates of Gauss in terms of p, t ?
3. You are given the x, y coordinates of a line $\mathbb{R}^2$ in terms of time t and a point $\vec{p} = (2, 3)$ in $\mathbb{R}^2$ and a vector $\vec{v} = (1, 2)$ as $\vec{p} + t\vec{v}$. Can you find the equation of this line? (Hint: try to eliminate t in terms of x, y .)

Takeaway: The above exercise teaches us that a line in $\mathbb{R}^2$ is dependent on one variable t and so is one dimensional.

Activity 2

1. You are given a point (x_0, y_0) in $\mathbb{R}^2$. If you want to uniquely determine this point, it is sufficient to give equations of a certain number of lines that this point passes through in order to determine what it is. How many lines are sufficient?
2. Equation of a plane looks like $ax + by + cz = d$. Given a point (x_0, y_0, z_0) in $\mathbb{R}^3$, how many distinct planes do you need to uniquely determine this point?

Problem 3

1. Turn the following system of equations into the form of an augmented matrix:

$$2x + 3y = 4 \tag{1}$$

$$10x + 15y = 20 \tag{2}$$

2. Do Jordan elimination of the above augmented matrix.
3. Does there exist a unique solution? If not, what is going wrong?

Problem 4

1. Do Gauss Jordan elimination of the following matrix:

$$\begin{pmatrix} 1 & 5 & 6 \\ 2 & 3 & 4 \\ 24 & 71 & 88 \end{pmatrix}$$

2. Consider the linear system

$$\begin{cases} x + 5y + 6z = 1, \\ 2x + 3y + 4z = 1, \\ 24x + 71y + 88z = 17. \end{cases}$$

Find the solution set of the system. Are they finite or infinite?

3. Consider the linear system

$$\begin{cases} x + 5y + 6z = 1, \\ 2x + 3y + 4z = 1, \\ 24x + 71y + 88z = 18. \end{cases}$$

Find the solution set of the system. Are they finite or infinite?

Takeaway: Jordan elimination and RREF help us determine whether linear systems are actually distinct or not (upto linear combination. If M is $n \times n$ matrix whose RREF has a zero row, then $M\vec{x} = \vec{a}$ either has no solutions or infinitely many solutions!!

Time distribution (Problem 1: 15 mins, Problem 2: 5 mins, Problem 3: 7 mins, Problem 4: 23 mins)

Solutions:
Activity 1:

1. Newton is on the line $y = 7x$, which passes through the origin. Choose any nonzero direction vector parallel to this line, for example

$$\vec{v} = (1, 7).$$

Then the plane's path starting at the origin is

$$(x(t), y(t)) = (0, 0) + t(1, 7) = (t, 7t),$$

which lies entirely on $y = 7x$, so you will find Newton.

2. Gauss is on the line $y = 7x - 23$, which does *not* pass through the origin. Choose a point on this line, for instance take $x = 0$, giving $y = -23$, so let

$$\vec{p} = (0, -23).$$

Choose a direction vector parallel to the line:

$$\vec{v} = (1, 7).$$

Then the plane's path is

$$\vec{p} + t\vec{v} = (0, -23) + t(1, 7) = (t, -23 + 7t),$$

which satisfies $y = 7x - 23$ for all t , so you will find Gauss.

If the time to reach Gauss from $\vec{p}$ is t , then the coordinates of Gauss are

$$\vec{G} = \vec{p} + t\vec{v}.$$

Writing $\vec{p} = (p_1, p_2)$ and $\vec{v} = (v_1, v_2)$, this is

$$\vec{G} = (p_1 + tv_1, p_2 + tv_2).$$

(In our concrete choice $\vec{p} = (0, -23)$ and $\vec{v} = (1, 7)$, we get $\vec{G} = (t, -23 + 7t)$.)

3. With $\vec{p} = (2, 3)$ and $\vec{v} = (1, 2)$, the parametric form is

$$(x, y) = (2, 3) + t(1, 2),$$

so

$$x = 2 + t, \quad y = 3 + 2t.$$

Eliminate t : from $x = 2 + t$ we get $t = x - 2$. Substitute into y :

$$y = 3 + 2(x - 2) = 3 + 2x - 4 = 2x - 1.$$

Therefore the equation of the line is

$$y = 2x - 1.$$

Problem 4:
Consider

$$\begin{cases} x + 5y + 6z = 1, \\ 2x + 3y + 4z = 1, \\ 24x + 71y + 88z = 17. \end{cases}$$

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 2 & 3 & 4 & 1 \\ 24 & 71 & 88 & 17 \end{array} \right].$$

Row reduce:

$$R_2 \leftarrow R_2 - 2R_1, \quad R_3 \leftarrow R_3 - 24R_1$$

gives

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & -7 & -8 & -1 \\ 0 & -49 & -56 & -7 \end{array} \right].$$

Next,

$$R_2 \leftarrow -\frac{1}{7}R_2$$

gives

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & -49 & -56 & -7 \end{array} \right].$$

Then

$$R_3 \leftarrow R_3 + 49R_2$$

gives

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Finally,

$$R_1 \leftarrow R_1 - 5R_2$$

gives the reduced row echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{7} & \frac{2}{7} \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Let $z = t$ be free. Then

$$x + \frac{2}{7}z = \frac{2}{7} \Rightarrow x = \frac{2}{7} - \frac{2}{7}t, \quad y + \frac{8}{7}z = \frac{1}{7} \Rightarrow y = \frac{1}{7} - \frac{8}{7}t.$$

So the solution set is

$$(x, y, z) = \left(\frac{2}{7}, \frac{1}{7}, 0 \right) + t \left(-\frac{2}{7}, -\frac{8}{7}, 1 \right), \quad t \in \mathbb{R}.$$

Hence there are infinitely many solutions.

Problem 2.

Problem 1 (infinitely many solutions)

System:

$$\begin{cases} x + 5y + 6z = 1, \\ 2x + 3y + 4z = 1, \\ 24x + 71y + 88z = 17 \end{cases}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 2 & 3 & 4 & 1 \\ 24 & 71 & 88 & 17 \end{array} \right]$$

$$R_2 \leftarrow R_2 - 2R_1, \quad R_3 \leftarrow R_3 - 24R_1$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & -7 & -8 & -1 \\ 0 & -49 & -56 & -7 \end{array} \right]$$

$$R_2 \leftarrow -\frac{1}{7}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & -49 & -56 & -7 \end{array} \right]$$

$$R_3 \leftarrow R_3 + 49R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_1 \leftarrow R_1 - 5R_2$$

$$\text{RREF: } \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{7} & \frac{2}{7} \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

 $z = t$ free:

$$x = \frac{2}{7} - \frac{2}{7}t, \quad y = \frac{1}{7} - \frac{8}{7}t$$

$$(x, y, z) = \left(\frac{2}{7}, \frac{1}{7}, 0 \right) + t \left(-\frac{2}{7}, -\frac{8}{7}, 1 \right).$$

Problem 2 (no solution)

System:

$$\begin{cases} x + 5y + 6z = 1, \\ 2x + 3y + 4z = 1, \\ 24x + 71y + 88z = 18 \end{cases}$$

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 2 & 3 & 4 & 1 \\ 24 & 71 & 88 & 18 \end{array} \right]$$

$$R_2 \leftarrow R_2 - 2R_1, \quad R_3 \leftarrow R_3 - 24R_1$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & -7 & -8 & -1 \\ 0 & -49 & -56 & -6 \end{array} \right]$$

$$R_2 \leftarrow -\frac{1}{7}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & -49 & -56 & -6 \end{array} \right]$$

$$R_3 \leftarrow R_3 + 49R_2$$

$$\text{RREF (shows inconsistency): } \left[\begin{array}{ccc|c} 1 & 5 & 6 & 1 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 \end{array} \right]$$

Last row is $0 = 1$, so there is no solution.