

Calculus Cheat Sheet: Differentiation, Integration, and Trigonometry

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1. Basic Differentiation Formulas

$$\begin{aligned}\frac{d}{dx}x^n &= nx^{n-1}, \quad n \in \mathbb{R} \\ \frac{d}{dx}e^x &= e^x, \quad \frac{d}{dx}a^x = a^x \ln a \\ \frac{d}{dx}\ln x &= \frac{1}{x}, \quad \frac{d}{dx}\log_a x = \frac{1}{x \ln a} \\ \frac{d}{dx}\sin x &= \cos x, \quad \frac{d}{dx}\cos x = -\sin x \\ \frac{d}{dx}\tan x &= \sec^2 x, \quad \frac{d}{dx}\cot x = -\csc^2 x \\ \frac{d}{dx}\sec x &= \sec x \tan x, \quad \frac{d}{dx}\csc x = -\csc x \cot x \\ \frac{d}{dx}\arcsin x &= \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\arctan x &= \frac{1}{1+x^2}, \quad \frac{d}{dx}x = -\frac{1}{1+x^2}\end{aligned}$$

Derivative Rules

- Linearity: $(af + bg)' = af' + bg'$
- Product Rule: $(uv)' = u'v + uv'$
- Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- Chain Rule: $(f(g(x)))' = f'(g(x))g'(x)$

2. Basic Integration Formulas

$$\begin{aligned}\int x^n dx &= \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \\ \int \frac{1}{x} dx &= \ln |x| + C \\ \int e^x dx &= e^x + C, \quad \int a^x dx = \frac{a^x}{\ln a} + C \\ \int \sin x dx &= -\cos x + C, \quad \int \cos x dx = \sin x + C \\ \int \sec^2 x dx &= \tan x + C, \quad \int \csc^2 x dx = -\cot x + C \\ \int \sec x \tan x dx &= \sec x + C, \quad \int \csc x \cot x dx = -\csc x + C \\ \int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin \frac{x}{a} + C \\ \int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \arctan \frac{x}{a} + C\end{aligned}$$

Integration Facts

- Linearity: $\int [af(x) + bg(x)]dx = a \int f(x)dx + b \int g(x)dx$
- Scaling: $\int f(cx)dx = \frac{1}{c}F(cx) + C$, if $\int f(x)dx = F(x) + C$

3. Trigonometry Formulas

Euler Mnemonic for Sum/Difference Formulas

Euler Trick: Recall $e^{ix} = \cos x + i \sin x$. Then: $-\sin(A+B)$ has $+\cos A \sin B$, $\sin(A-B)$ has $-\cos A \sin B$. $-\cos(A+B)$ has $-\sin A \sin B$, $\cos(A-B)$ has $+\sin A \sin B$.

Angle Sum/Difference Formulas

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(A-B) &= \cos A \cos B + \sin A \sin B\end{aligned}$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x, \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

Power-Reduction Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

Product-to-Sum Formulas

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$